International Journal of Research in Engineering and Science (IJRES)
ISSN (Online): 2320-9364, ISSN (Print): 2320-9356
www.ijres.org Volume 4 Issue 10 ǁ October. 2016 ǁ PP. 81-88

Secure Complementary Equivalence Domination

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Abstract. Let $G=(V,E)$ be a simple finite undirected graph. A subset S of V(G) is called an equivalence set if every component of the induced sub graph $\langle S \rangle$ is complete. A graph G is an equivalence graph if every component of G is complete. A sub set S of V(G) is called a complementary equivalence dominating set of G if $\langle V-S \rangle$ is an equivalence set of G and S is a dominating set of G. The minimum cardinality of a c-e-d set of G is denoted by $\gamma_{c-e}(G)$. A subset S of V(G) is called a secure complementary equivalence dominating set of G if S is a complementary equivalence dominating set, and for any $v \in V-S$ there exist $u \in S$ such that $(S-\{u\}) \cup \{v\}$ is a complementary equivalence dominating set of G. The minimum cardinality of a secure complementary equivalence dominating set of G is called the secure complementary equivalence domination number of G and is denoted by $\gamma_{se}^{c-e}(G)$. In this paper, complementary equivalence domination is combined with security. Several results concerning secure complementary equivalence domination are derived. Further proper color partition and equivalence class partition may also be combined with security. Results involving these concepts are also derived.

Keywords: Complementary equivalence domination, Secure domination, Secure complementary equivalence domination.

I. INTRODUCTION.

E.J.Cockayne et al [5]introduced the concept of secure domination. A sub set S of G is called a secure dominating set of G if S is a dominating set of G and for any vertex $u \in V-S$, there exists a vertex $v \in S$, such that $(S-\{v\}) \cup \{u\}$)is a dominating s. A subset S of V(G) is called an equivalence set if every component of the induced sub graph $\langle S \rangle$ is complete. A graph G is an equivalence graph if every component of G is complete. A sub set S of V(G) is called a complementary equivalence dominating set of G if $\langle V-S \rangle$ is an equivalence set of G and S is a dominating set of G. The minimum cardinality of a c-e-d set of G is denoted by $\gamma_{c-e}(G)$. A subset S of V(G) is called a secure complementary equivalence dominating set of G if S is a complementary equivalence dominating set, and for any $v \in V-S$ there exist $u \in S$ such that $(S-\{u\}) \cup \{v\}$ is a complementary equivalence dominating set of G. The minimum cardinality of a secure complementary equivalence dominating set of G is called the secure complementary equivalence domination number of G and is denoted by $\gamma_{se}^{c-e}(G)$. In this paper, complementary equivalence domination is combined security. Several results concerning secure complementary equivalence domination are derived. Further proper color partition and equivalence class partition may also be combined with security. Results involving these concepts are also derived.

II. SECURE COMPLEMENTARY EQUIVALENCE DOMINATION

Definition 7.1. Let G = (V,E) be a simple finite undirected graph. A subset S of V(G) is called a secure complementary equivalence dominating set of G if S is a complementary equivalence dominating set, and for any $v \in V-S$ there exist $u \in S$ such that $(S-\{u\}) \cup \{v\}$ is a complementary equivalence dominating set of G. The minimum cardinality of a secure complementary equivalence dominating set of G is called the secure complementary equivalence domination number of G and is denoted by $\gamma_{se}^{c-e}(G)$.

Secure complementary equivalence of domination set is super hereditary.

Example 2.2.

1. Let $G = K_n$. Any single vertex K_n is a secure complementary equivalence dominating set and hence $\gamma_{se}^{c-e}(K_n) = 1$.

2. Let $G = K_{1,n}$. The set containing the central vertex and pendent vertex of $K_{1,n}$ is a secure complementary equivalence dominating set and hence $\gamma_{se}^{c-e}(K_{1,n}) = 2$.

3. Let $G = K_{m,n}$. Let $m \le n$. Let V_1 and V_2 be the partite sets of G. Let $|V_1| = m$ and $|V_2| = n$.

Let $V_1 = \{u_1, u_2, \ldots, u_m\}$ and $V_2 = \{v_1, v_2, \ldots, v_n\}$. Let $S = V_1 \cup \{v_1\}$. Then S is a dominating set, $V - S = \{v_1, v_2, \ldots, v_n\}$ is an equivalence set. Therefore S is a complementary equivalence dominating set.

For any v_i $2 \le i \le n$ in V-S, there exist v_1 in S such that $(S - \{v_1\}) \cup \{v_i\} = V_1 \cup \{v_i\}$ is a complementary equivalence dominating set. Therefore S is a secure complementary equivalence dominating set.

$|S| = m + 1.$

Clearly S is of minimum cardinality.

Therefore $\gamma_{se}^{c-e}(K_{m,n}) = m + 1 = \min\{m, n\} + 1$

4. Let $G = C_n$. Then

$$\gamma_{se}^{c-e}(C_n) = \begin{cases} \frac{n}{2} \text{ if } \mathrm{n} \text{ is even} \\ \frac{\mathrm{n}-1}{2} \text{ if } \mathrm{n} \text{ is odd}, \mathrm{n} \ge 7 \\ 3 \text{ if } \mathrm{n} = 5 \end{cases}$$

5. Let $G = P_n$. Then

$$\gamma_{se}^{c-e}(P_n) = \begin{cases} \frac{n}{2} \text{ if } \mathrm{n} \text{ is even} \\ \frac{\mathrm{n}+1}{2} \text{ if } \mathrm{n} \text{ is odd}, \mathrm{n} \ge 7 \\ 2 \text{ if } \mathrm{n} = 5 \end{cases}$$

6. Let $G = W_n$. Then

$$\gamma_{se}^{c-e}(W_n) = \begin{cases} \frac{n-1}{2} \text{ if } \mathrm{n} \text{ is odd}, \mathrm{n} \ge 7 \\ \frac{\mathrm{n}}{2} \text{ if } \mathrm{n} \text{ is even} \\ \text{ if } \mathrm{n} = 5 \\ 1 \text{ if } \mathrm{n} = 4 \end{cases}$$

Remark 2.3. Since secure complementary equivalence dominating set is super hereditary, a secure complementary equivalence dominating set S is minimal iff $S - \{u\}$ is not a secure complementary equivalence dominating set for every $u \in S$.

Theorem 2.4. Let S be a secure complementary equivalence dominating set . S is minimal iff for every $u \in S$ one of the following holds.

i). $pn[u,S] \neq \phi$.

ii). $(V-S) \cup \{u\}$ is not component wise complete.

iii). For some $v \in (V-(S-\{u\}))$,there exist no $w \in S-\{u\}$ such that $(S-\{u\}-\{w\}) \cup \{v\}$ is a complementary equivalent dominating set.

Proof. Suppose condition (i) holds.

Then $S-\{u\}$ is not a dominating set. Suppose condition (ii) holds. Then $S-\{u\}$ is not a complementary equivalence set. Suppose condition (iii) holds. Then $S-\{u\}$ is not a secure complementary equivalence dominating set. Therefore S is a minimal complementary equivalence dominating set.

Conversely, Suppose S is a minimal secure complementary equivalence dominating set. Then for any $u \in S$, $S-\{u\}$ is not a secure complementary equivalence dominating set. If $S-\{u\}$ is not a dominating set then $pn[u,S] \neq \phi$.

Therefore (i) holds.

Suppose $S-\{u\}$ is a dominating set but not a complementary equivalence set. Then $(V-S) \cup \{u\}$ is not an equivalence set. Therefore (ii) holds.

Suppose $S-\{u\}$ is a complementary equivalence dominating set. Then $S-\{u\}$ is not a secure complementary equivalence dominating set. That is for some $v \in V-(S-\{u\})$ there exists no $w \in S-\{u\}$ such that $(S-\{u\}-\{w\}) \cup \{v\}$ is not a complementary equivalence dominating set. That is condition (iii) holds.

Theorem 2.5. A maximal independent set whose complement is an equivalence set is a complementary equivalence dominating set.

Proof. Let S be a maximal independent set such that V-S is an equivalence set. Let $v \in V-S$.

Suppose v is not dominated by S. Then $S \cup \{v\}$ is independent and $V-(S \cup \{v\})$ is component wise complete. This contradicts the fact that S is a maximal independent set with V-S being an equivalence set. Therefore S is a dominating set whose complement is an equivalence set. That is S is a Complementary equivalence dominating set.

Remark 2.6. A maximal independent set whose complement is an equivalence set need not be a secure dominating set. For example, when $G = C_5$ with $V(G) = \{u_1, u_2, u_3, u_4, u_5\}$, $\{u_1, u_3\}$ is a maximal independent set whose complement is an equivalence set. But $\{u_1, u_3\}$ is not a secure dominating set.

Theorem 2.7. $\gamma_{se}^{c-e}(G) = 1$ iff $G = K_n$

Proof. Let $\gamma_{se}^{c-e}(G) = 1$. Then there exist a vertex u such that {u} is a dominating set. Therefore {u} is adjacent with every vertex of G-{u}. Since {u} is a secure dominating set, for any $v \in V-\{u\}$, {v} is a dominating set. That is v is adjacent to every vertex of G-{v}. Therefore every vertex of G is a full degree vertex. That is G is complete.

The converse is obvious.

Definition 2.8. If S is a subset of V(G) and if for any $u \in S$ if u satisfies conditions (i), (ii) or (iii) of Theorem 7.4, then S is called a sce-irredundant set of G.

Theorem 2.9. sce-irredundance is hereditary.

Proof. Let S be a subset of V(G) such that S is sce-irredundant. Let T be a subset of S, $T \neq S$. Let $x \in T$. Therefore $x \in S$. Suppose x satisfies (i). Then $pn[x,S] \neq \varphi$. That is either x is an isolate of S or x has a private neighbour in V-S. Therefore x is an isolate of T or x has a private neighbour in V-T. Therefore $pn[x,S] \neq \varphi$. Suppose x satisfies (ii), that is $(V-S) \cup \{x\}$ is not component wise complete. Suppose $(V-T) \cup \{x\}$ is component wise complete. Then $(V-S) \cup \{x\}$ being a subset of $(V-T) \cup \{x\}$

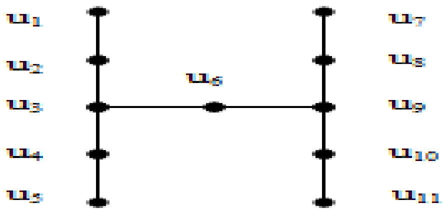
is component wise complete, a contradiction. Therefore $(V-T)\cup\{x\}$ is not component wise complete. Suppose x satisfies condition (iii). That is for some $v\in(V-(S-\{x\}))$ there exists no w in S-{x} such tht $(S-\{x\}-\{w\})\cup\{v\}$ is a equivalence dominating set.

Since $T\underset{\neq}{\subset}S$, $(V-(T-\{x\}))\supset(V-(S-\{x\}))$. Therefore $v\in(V-(T-\{x\}))$. Since there exists no w in S-{x}, such that $(S-\{x\}-\{w\})\cup\{v\}$ is a complementary equivalence dominating set of G, $w\notin T-\{x\}$. $(T-\{x\}-\{w\})\cup\{v\}$ is not a dominating set of G. Suppose $(S-\{x\}-\{w\})\cup\{v\}$ is not a complementary equivalence set, then $V-((S-\{x\}-\{w\})\cup\{v\})$ is not component wise complete. Then $V-((T-\{x\}-\{w\})\cup\{v\})$ is not component wise complete. For otherwise $V-((S-\{x\}-\{w\})\cup\{v\})$ being a subset of $V-((T-\{x\}-\{w\})\cup\{v\})$ is component wise complete, a contradiction. Therefore, for some $v\in V-(T-\{x\})$, there exists no $w\in T-\{x\}$ such that $(T-\{x\}-\{w\})\cup\{v\}$ is a complementary equivalence dominating set. Hence sce-irredundance is hereditary.

Remark 2.10. A sce-irredundant set is maximal if and only if it is 1-maximal.
Definition 2.11. The minimum (maximum) cardinality of a maximal sce-irredundant set of G is called the minimum(maximum) sce-irredundance number of G and is denoted by ir-sce(G) (IR-sce(G)).
Theorem 2.12. A minimal secure complementary equivalence dominating set of G is a maximal sce-irredundant set of G.

Proof. Let S be a minimal secure complementary equivalence dominating set of G. Therefore S is a sce-irredundant set of G. Suppose S is not a maximal sce-irredundanr set of G. Therefore there exists a vertex v in V-S such that $S\cup\{v\}$ is a sce-irredundant set of G. Therefore either $pn[v,S\cup\{v\}]\neq\varphi$, in which case, there exists a vertex $w\in V-(S\cup\{v\})$ such that w is a private neighbour of v with respect to $S\cup\{v\}$. That is either v is an isolate of S or w is adjacent with v only in $S\cup\{v\}$. That is w is not adjacent with any vertex of S, a contradiction, since S is a dominating set. Suppose $V-((S\cup\{v\})\cup\{u\}$ is not component wise complete where $u\in S\cup\{v\}$. Therefore $V-((S\cup\{v\})\cup\{v\}$ is not componentwise complete. That is V-S is not componentwise complete, a contradiction. Since S is a complementary equivalence set of G. Suppose for some $w\in V-((S\cup\{v\})-\{u\})$, there exists no $z\in(S\cup\{v\})-\{u\}$ such that $(S\cup\{v\}-\{u\}-\{z\})\cup\{w\}$ is complementary equivalence dominating set where $u\in S\cup\{v\}$. Taking u = v, we get that for some $w\in V-S$, there exists no $z\in S$ such that $(S-\{z\})\cup\{w\}$ is a complementary equivalence dominating set of G. Therefore S is a maximal sce-irredundant set of G.

Remark 2.13. From the above theorem, we get , $ir_{sce}(G)\leq\gamma_{se}^{c-e}(G)\leq\Gamma_{se}^{c-e}(G)\leq IR_{sce}(G)$.
Remark 2.**14**.



G

Figure 7.1

Let $S=\{u_2,u_4,u_6,u_8,u_{10}\}$. S is a secure equivalence dominating set of G. S is also a minimal secure complementary equivalence dominating set of G. Therefore S is a maximal sce-irredundant set.
Let $T=\{u_2,u_3,u_8,u_9\}$. Clearly T is a maximal sce-irredundant set of G. Therefore $ir_{sce}(G)\le 4$ and $\gamma_{se}^{c-e}(G)=5$. Therefore $ir_{sce}(G)\prec\gamma_{se}^{c-e}(G)$.

III. SECURE EQUIVALENCE CHROMATIC PARTITION

Definition 3.1. A partition $\Pi=\{V_1,V_2,...,V_k\}$ is called a secure equivalence partition of G is each V_i is a secure equivalence set of G. This partition is briefly called se-partition of G.

Remark 3.12. If $V(G)=\{u_1,u_2,...,u_n\}$ then $\Pi=\{\{u_1\},\{u_2\},...,\{u_n\}\}$ is a secure equivalence partition of G.
secure equivalence chromatic number of G and is denoted by $\chi_{se}(G)$.

$\chi_{se}(G)$ **for standard graphs:**

1. $\chi_{se}(K_n)=1$.
2. $\chi_{se}(\overline{K_n})=1$.
3. $\chi_{se}(G)=1$ if G is an equivalence graph.
4. $\chi_{se}(K_{1,n})=\begin{cases}\dfrac{n+1}{2} \text{ if n is odd}\\ \dfrac{\text{n}}{2}+1 \text{ if n is even}\end{cases}$

5. Let $m\le n$. Then $\chi_{se}(K_{m,n})=\begin{cases}\dfrac{m+n+1}{2} \text{ if m and n are of opposite parities}\\ \dfrac{\text{m+n}}{2} \text{ if m and n are of the same parity}\end{cases}$

6. $\chi_{se}(P_n)=2$

Proof.If $V(P_n)=\{u_1,u_2,u_3,...,u_n\}$ then
$\Pi=\{\{u_1,u_2,u_5,u_6,...,u_{n-3},u_{n-2}\},\{u_3,u_5,u_7,u_8,u_{11},u_{12},...,u_{n-1},u_n\}$ is a se-partition of P_n if n is even, and $\Pi=\{\{u_1,u_2,...,u_{n-2},u_{n-1}\},\{u_3,u_4,...,u_{n-4},u_{n-3},u_n\}$ is se-partition of P_n if n is odd.

7. $\chi_{se}(C_n)=2$

Proof.If $V(C_n)=\{u_1,u_2,u_3,...,u_n\}$ then
$\Pi=\{\{u_1,u_2,u_4,u_6,...,u_{n-3},u_{n-2}\},\{u_3,u_5,u_7,u_8,u_{11},u_{12},...,u_{n-1},u_n\}$ is a se-partition if n is even, and $\Pi=\{\{u_1,u_2,u_4,u_6,...,u_{n-2},u_{n-1}\},\{u_3,u_5,u_7,u_8,u_{11},u_{12},...,u_n\}$ is se-partition if n is odd.

8. $\chi_{se}(W_n) = \begin{cases} 1 \text{ if } n = 4 \\ 2 \text{ if } n = 5 \\ 3 \text{ if } n \geq 6 \end{cases}$

9. $\chi_{se}(D_{r,s}) = \left\lceil \frac{r}{2} \right\rceil + \left\lceil \frac{s}{2} \right\rceil + 1$

10. $\chi_{se}(P) = 3$ where P is the Petersen graph.

Remark 3.13 $\chi_{se}(G) = 1$ if and only if G is an equivalence graph.

Theorem 3.14 $\chi_{se}(G) < n$ for any graph G of order $n \succ 1$.

Proof. Suppose G has k isolates, $k \geq 2$. Then any set of two isolates form a secure equivalence set and hence $\chi_{se}(G) \prec n$. Su1ppose G has exactly one isolate. Let $n \geq 3$. Then the other vertices of G(of cardinality greater than or equal to 2) has a K_2 and hence $\chi_{se}(G) \prec n$. If n = 2 then $\chi_{se}(G) = 1$. Suppose G has no isolates. Let $n \geq 3$. Then G contains a K_2 which is a secure equivalence set of G. Therefore $\chi_{se}(G) \prec n$. If n = 2 then $\chi_{se}(G) = 1$.

Theorem 3.15. $\chi_{se}(G) = n-1$ if and only if $G = P_3, K_2 \cup K_1, K_2, \overline{K_2}$.

Proof. Suppose $\omega(G) = r \geq 3$. Then $\chi_{se}(G) \prec n-1$. Therefore $\chi_{se}(G) = n-1$, then $\omega(G) \leq 2$. If $\beta_0(G) = r \geq 3$ then $\chi_{se}(G) \prec n-1$. Therefore if $\chi_{se}(G) = n-1$ then $\beta_0(G) \leq 2$. If $\omega(G) = 2$ then $G = P_3$ or $K_2 \cup K_1$. If $\omega(G) = 1$ then $G = \overline{K_n}$. Then $\chi_{se}(G) = n-1$ if n = 2. Therefore if $\chi_{se}(G) = n-1$ then $G = P_3, K_2 \cup K_1, K_2, \overline{K_2}$
The converse is obvious.

Remark 3.16. Let G be a graph with $\chi_{se}(G) = k$. Let $\Pi = \{V_1, V_2, ..., V_k\}$ be a secure equivalence partition of G. If V_i and V_j do not have any edge between them, then $V_i \cup V_j$ is an equivalence class which is also secure. Therefore $\chi_{se}(G) \leq k-1$, a contradiction. Therefore for any i,j, $i \neq j, 1 \leq i, j \leq k$, V_i and V_j have an edge between them.

Remark 3.17. Let $\Pi = \{V_1, V_2, ..., V_k\}$ be a secure equivalence partition of G of cardinality $\chi_{se}(G)$. Suppose every vertex of V_i is not adjacent with some v_j, $j \neq i$. Then the vertices of V_i can be attached to other classes which also remain secure after attachment. Hence we get $\chi(G) < k$, a contradiction. Therefore there exist a vertex of V_i which is adjacent with every class V_j, $i \neq j, 1 \leq i, j \leq k$.

IV. SECURE CHROMATIC PARTITION

Definition 4.1 A secure proper vertex color partition of a graph G is a partition $\Pi = \{V_1, V_2, ..., V_k\}$ where each V_i is independent and secure. The minimum cardinality of such a partition is called secure chromatic number of G and is denoted by $\chi_s(G)$.

Clearly $\chi(G) \leq \chi_s(G)$.

Remark 4.2.

i). $\chi_s(K_n) = n$.

ii). $\chi_s(\overline{K_n}) = 1$

iii). $\chi_s(K_{1,n}) = n+1$.

iv) $\chi_s(K_{m,n}) = m+n$

v) $\chi_s(P_n) = 3$

Proof. Let $V(P_n) = \{u_1, u_2, u_3, ..., u_n\}$.

when n =3k, $\{u_1, u_4, u_7, ..., u_{3k-2}\}, \{u_2, u_5, u_8, ..., u_{3k-1}\}\{u_3, u_6, u_9, ..., u_{3k}\}$ is a secure chromatic partition of P_n.

When n = 3k+1, $\{u_1, u_4, u_7, ..., u_{3k+1}\}, \{u_2, u_5, u_8, ..., u_{3k-1}\}\{u_3, u_6, u_9, ..., u_{3k}\}$ is a secure chromatic partition of P_n.

When n = 3k+2, $\{u_1, u_4, u_7, ..., u_{3k+1}\}, \{u_2, u_5, u_8, ..., u_{3k+2}\}\{u_3, u_6, u_9, ..., u_{3k}\}$ is a secure chromatic partition of P_n.

vi) $\chi_s(C_n) = \begin{cases} 3 \text{ if } n \equiv 0 (\text{mod } 3), n \geq 3 \\ 4 \text{ if } n \equiv 1 (\text{mod } 3), n \geq 4 \\ 5 \text{ if } n \equiv 2 (\text{mod } 3), n \geq 8 \end{cases}$.

vii) $\chi_s(C_5) = 4$.

viii) $\chi_s(W_n) = n$

ix) $\chi_s(P) = 10$ where P is the Petersen graph.

x) $\chi_s(D_{r,s}) = s + 2 \text{ if } r \leq s$.

Remark 4.3. $\chi_{se}(G) \leq \chi(G)$. That is $\max\{\chi(G), \chi_{se}(G)\} \leq \chi_s(G)$.

Observation 4.4

i. Any two elements of a χ_s partition of G have an edge between them.

ii. In a χ_s-partition, every class contains an element which is adjacent with every other class of the partition.

Remark 4.5 $\chi(G) \leq \chi_s(G) \leq \psi_b(G), \chi(G) \leq \chi_s(G) \leq \psi(G)$.

Remark 4.6. Any χ_s-partition of G can be modified into a χ_s-partition such that there exists a class which is a maximal independent dominating set of G.

Theorem 4.7. If a and b are positive integers such that $2 \leq a \leq b$ then there exists a graph G such that $\chi(G) = a$ and $\chi_{se}(G) = b$.

Proof. Given $a \geq 2$. Let $G = K_a \cup K_{b-1,b-1}$.

$\chi(G) = a$ and $\chi_{se}(G) = 1 + b - 1 = b$.

Remark 4.8. If a =1, then G is $\overline{K_n}$ for some n in which case $\chi_{se}(G) = 1$.

Observation 4.9. If G is a cycle of even order then $\chi(G) = \chi_{se}(G) = 2$ .If G is an odd cycle then $\chi(G) = \chi_{se}(G) + 1$.

Observation 4.10. There is no relationship between $\chi(G)$ and $\chi_{se}(G)$ for when $n \geq 2$.

$\chi(K_n) = n$ and $\chi_{se}(K_n) = 1$ .when n is odd, $\chi(C_n) = 3$ and $\chi_{se}(C_n) = 2$.

When m and n are even and $m + n \geq 6$ then $\chi(K_{m,n}) = 2$ and $\chi_{se}(K_{m,n}) = \frac{m+n}{2} \geq 3$.

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